Unit-V

0. A two-hinged parabolic arch with moment of inertia varying such that $I = I_c \sec\theta$, θ being the slope of the arch axis, has a span of 40 m and a central rise of 6 m. A point load of 60 kN acts at a section 12 m to the right of the left support. Calculate the horizontal reaction and also find the value and nature of bending moment under the load.

1. (a) State and explain the Muller-Breslau Principle. 2

(b) Compute the ordinates, at interval of 1·25 m, of the IL for the moment at A in the propped cantilever beam as shown in Fig.-8. The flexural rigidity is constant throughout. The propped cantilever shown in figure is subjected to 100 kN at mid span, making use of the influence lime ordinates compute the BM at A.

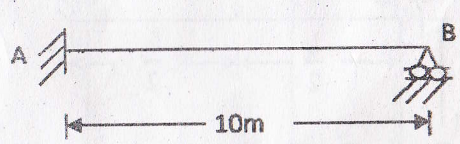


Fig.-8

mjams

Printed Pages– 10

Centre Supdt.
Centre No. - 84
I. T., G.G.V. Bilaspur [C.G.]

AS-4137

B. Tech. (Fifth Semester) Examination, 2013

[Civil Engg. (I.T.)]

STRUCTURAL ANALYSIS-II

(31CE01T)

Time Allowed : Three hours

Maximum Marks : 60

Note : *Attempt section-A which is compulsory and carries 20 marks. Section-B carries 40 marks in which each question carries 8 marks. Attempt **five** questions from section-B in which **one** question from each UNIT is compulsory. Missing data, if any, may be assumed suitably.*

Section-A

(Objective Type Questions) 10×2=20

Note : *Attempt all questions. Each question carries 2 marks.*

1. Write the correct alternative or answer (as the case may be) in the following (i) to (x) :

(i) A propped cantilever beam is loaded by a concentrated load W at its propped support. The reaction at the prop is :

(a) 3W/8

(b) W

(c) W/2

(d) 5W/8

(ii) It is difficult to use the strain energy method if for a structure :

(a) degrees of freedom are less

(b) degrees of freedom are more

(c) degrees of redundancy are less

(d) degrees of redundancy are more

(iii) If the hinged end of a propped cantilever (span and flexural rigidity EI) undergoes a rotation (θ) then the shear in the beam will be :

(a) $EI\,\theta/L^2$

(b) $2EI\,\theta/L^2$

(c) $3EI\,\theta/L^2$

(d) $6EI\,\theta/L^2$

(iv) Clockwise moment M are acting at both the enc of a uniform simply supported beam. The ratio c slope at the end to the slope at centre will be :

(a) 0·5

(b) 1

(c) 2

(d) 3

(v) The stiffness factor at the near end of a membe with far end hinged is :

(a) $4EI/L$

(b) $3EI/L$

(c) EI/L

(d) EI

(vi) The distribution factr of a member at a joint is :

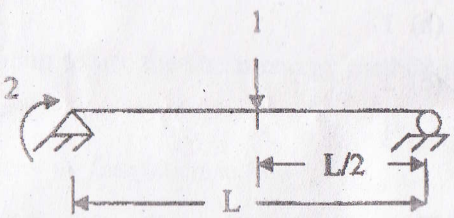
(a) the ratio of the moment borne by the member to the total moment applied at the joint

(b) the ratio of the area of the member to the sum of the areas of several members

(c) the ratio of the moment induced at the far end to the moment applied at the near end

(d) None of the above

(vii) The flexibility coefficient f_{22} for the beam shown in fig. below is :



(a) L/EI

(b) $L/2EI$

(c) $L/3EI$

(d) $L/4EI$

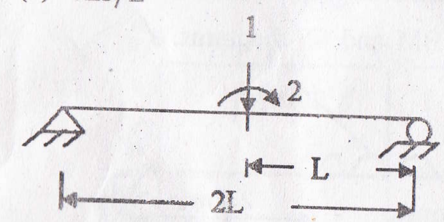
(viii) The stiffness coefficient k_{12} for the beam shown in Fig. below is :

(a) $-6EI/L^2$

(b) $-3EI/L^2$

(c) $3EI/L^2$

(d) $6EI/L^2$



(ix) For a two hinged arch if one of the supports settle down vertically, then the horizontal thrust :

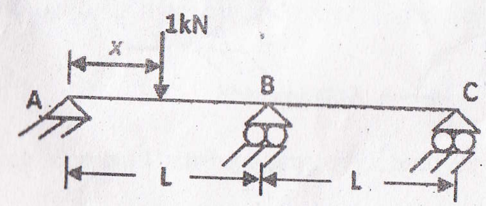
(a) is increased

(b) is decreased

(c) remains unchanged

(d) None of these

(x) For the continuous beam shown in fig. below, draw the qualitative influence line diagram for SF at any section between supports B and C.



P

Section-B

Unit-I

2. A beam of length 5m is loaded as shown in Fig.-1. Using consistent deformation method, calculate the reaction at B. Also draw BM and SF diagrams.

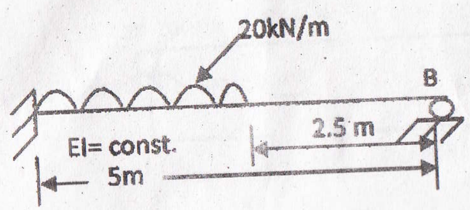


Fig.-1

3. Using principle of least work, determine the support reactions at C for the frame shown in Fig.-2. The EI values are indicated along the members.

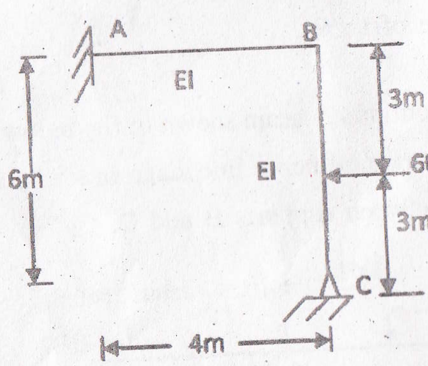


Fig.-2

Unit-II

4. Using slope deflection method, determine the end moments for the frame shown in Fig.-3 and hence draw the BM diagram. EI is constant for all the members.

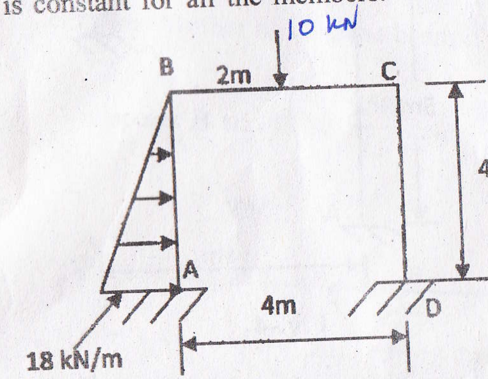


Fig.-3

5. A beam ABCD is fixed at ends A and simply supported at intermediate points B and C, the other end D being kept free. The lengths AB, BC and CD are 6 m, 8 m and 2 m respectively. The beam carries a u.d.l. of 30 kN/m acting on span BC and the overhang CD. The flexural rigidity is EI for span AB and is 2 EI for the rest of the beam. Using slope deflection method determine the end moments and hence draw bending moment and shear force diagrams.

Unit-III

6. Using Moment Distribution method, analyze the frame shown

PTO

Fig. 4. EI is constant for all the members. Also draw BM
agram.

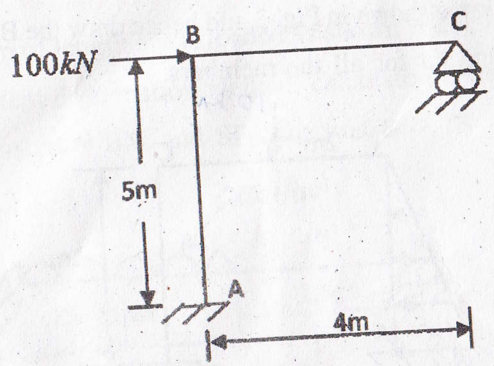


Fig.-4

Analyse the frame shown in fig.-5 using moment distribution method, if end A is subjected to anticlockwise rotation 0·002 radians and also it sinks down by 6 mm. Take EI=5×10^{10} kN mm^2 as constant for all the members. Also draw the BM diagram.

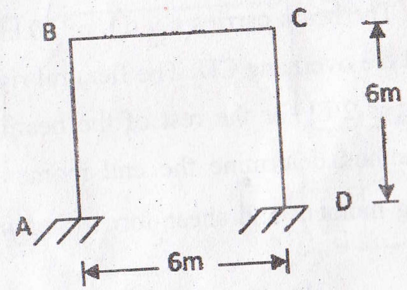


Fig.-5

Unit-IV

8. Using flexibility matrix method determine the end moments for the continuous beam ABC shown in Fig.-6. The EI is constant. Further analyze the beam if it undergoes settlement of supports B and C by $\frac{350}{EI}$ and $\frac{250}{EI}$ respectively.

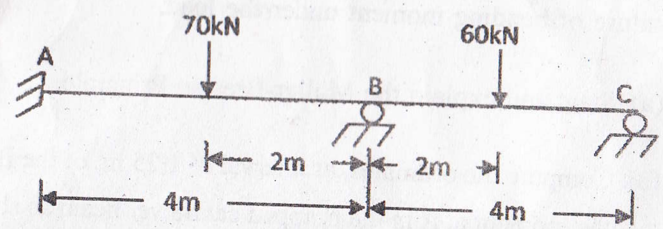


Fig.-6

9. Using stiffness matrix method analyze the rigid frame shown in Fig.-7. Neglect axial deformation and consider EI is constant throughout.

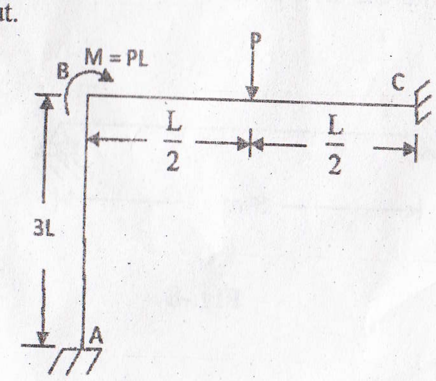


Fig.-7

MODEL SOLUTION

AS-4137

B. Tech. (Fifth Semester) Exam. 2013

Civil Engg. (I.T.)

Structural Analysis - II (31CE01T)

SECTION - A

Q1.

(i) — (b)

(ii) — (d)

(iii) — (d)

(iv) — (c)

(v) — (b)

(vi) — (a)

(vii) — (c)

(viii) — (b)

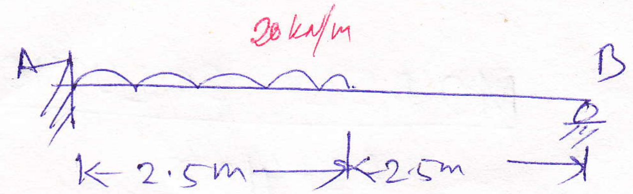
(ix) — (b)

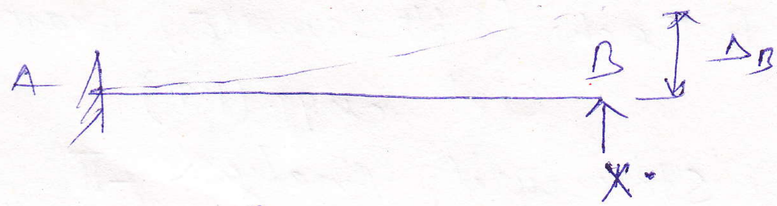
(x)

ILD for F_D

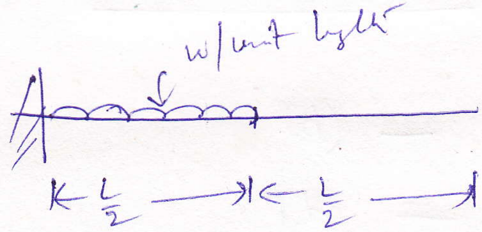
(02)

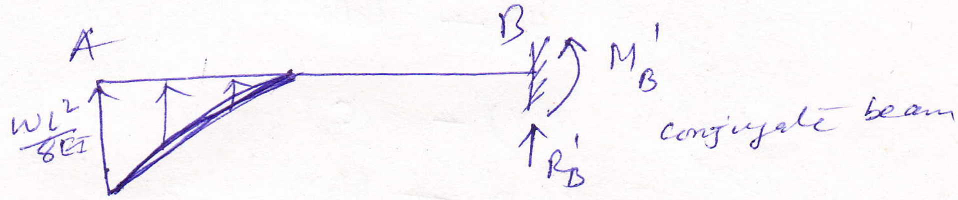
Q2





$$\Delta_B = \frac{X \cdot 5^3}{3EI} \uparrow$$





Area of load

$$= \frac{1}{3} * \left(\frac{wl^2}{8}\right) * \frac{l}{2} = \frac{wl^3}{48EI}$$

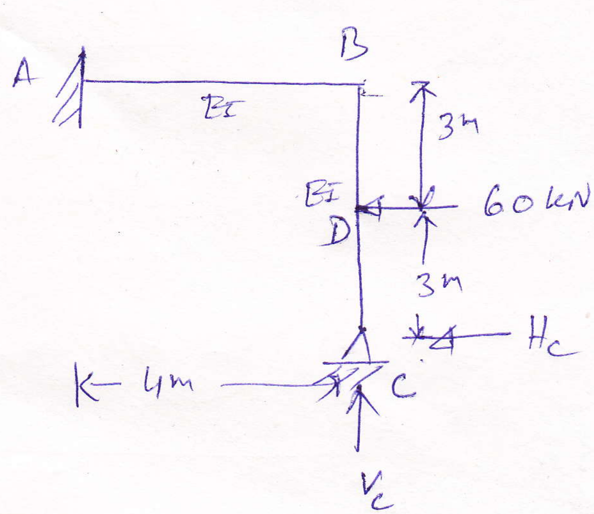
C.G. of load from B. $\left(\frac{l}{2} + \frac{3}{4} * \frac{l}{2}\right)$

$$= \frac{l}{2} + \frac{3l}{8} = \frac{(4+3)l}{8} = \frac{7l}{8}$$

$$\delta_B' = \frac{wl^3}{48EI} * \frac{7l}{8} = \frac{7wl^4}{384EI}$$

$$\therefore \quad X \cdot \frac{l^3}{3\cancel{EI}} = \frac{7wl^4}{384\cancel{EI}}$$

Q 3



V_c & H_c are redundant forces

$V_c = X_1$

$H_c = X_2$

Section	origin	Limit (m)	M_x	$\frac{\partial M}{\partial X_1}$	$\frac{\partial M}{\partial X_2}$
BA	B	0–4	$X_1 \cdot x - X_2 \cdot 6 - 60 * 3$	x	-6
CD	C	0 – 3	$-X_2 \cdot x$	0	$-x$
DB	C	3 – 6	$-X_2 \cdot x - 60(x-3)$	0	$-x$

$$\frac{\partial U}{\partial X_1} = 0 \Rightarrow \int_0^L M \frac{\partial M}{\partial X_1} \frac{dx}{EI} = 0$$

$$\Rightarrow \int_0^4 (X_1 \cdot x - X_2 \cdot 6 - 180) \cdot x\, dx + \int_0^3 (-X_2 \cdot x)\, 0 \cdot dx$$

$$+ \int_3^6 (-X_2 \cdot x - 60x + 180)\,(0)\, dx = 0$$

$$\int_0^4 (X_1 x^2 - 6X_2 x - 180x)\, dx = 0$$

$$\Rightarrow X_1 (21 \cdot 33) - 48 X_2 - 1440 = 0 \quad \cdots (1)$$

$\frac{\partial U}{\partial X_2} = 0 \Rightarrow \int_0^L M \frac{\partial M}{\partial X_2} \frac{dx}{EI} = 0$

$\Rightarrow \int_0^4 (X_1 \cdot x - 6X_2 - 180)(-6)\,dx + \int_0^3 X_2 x^2\,dx$

$+ \int_3^6 [X_2 \cdot x^2 + 60x(x-3)]\,dx = 0$

$\Rightarrow \int_0^4 (-6X_1 x + 36X_2 + 1080)\,dx + \int_0^3 X_2 x^2\,dx$

$+ \int_3^6 (X_2 x^2 + 60x^2 - 180x)\,dx$

$\Rightarrow -6X_1(8) + 144X_2 + 4320 + 9X_2$

$+ 63X_2 + 3780 - 2430 = 0$

$\Rightarrow -48X_1 + 216X_2 + 5670 = 0 \quad -(2)$

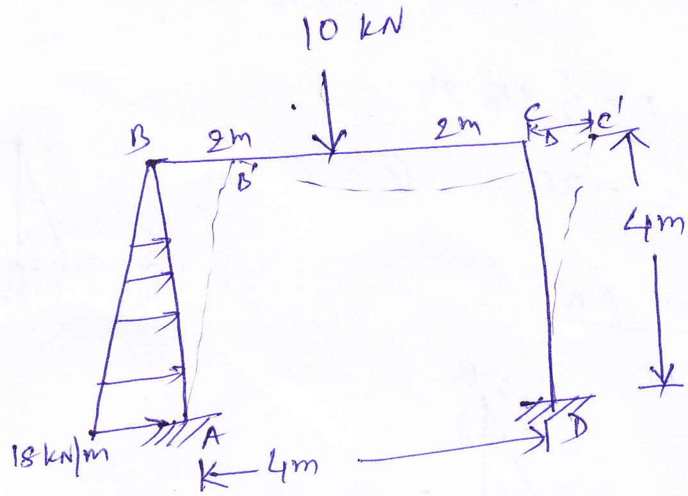
Multiply Eq(1) × 4.5

$95.99X_1 - 216X_2 - 6480 = 0$

$47.99X_1 = 810 \Rightarrow X_1 = 16.88\text{ kN}$

$X_2 = -22.50\text{ kN}$ ✓

Q4.



$$\left.\begin{aligned} M^F_{AB} &= -\frac{18\times 4^2}{20} = -14.4 \\ M^F_{BA} &= +\frac{18\times 4^2}{30} = +9.6 \\ M^F_{BC} &= -\frac{10\times 4}{8} = -5 \\ M^F_{CB} &= +\frac{10\times 4}{8} = +5 \end{aligned}\right\} \text{kNm}$$

$$M^F_{CD} = M^F_{DC} = 0$$

$$M_{AB} = -14.4 + \frac{2EI}{4}\left(0 + \theta_B - \frac{3\Delta}{4}\right)$$

$$= \frac{EI}{2}\left(\theta_B - \frac{3\Delta}{4}\right) - 14.4$$

$$M_{BA} = 9.6 + \frac{EI}{2}\left(2\theta_B - \frac{3\Delta}{4}\right)$$

$$M_{BC} = -5 + \frac{2EI}{4}\left(2\theta_B + \theta_C\right) = \frac{EI}{2}\left(2\theta_B + \theta_C\right) - 5$$

$$M_{CB} = 5 + \frac{EI}{2}\left(\theta_B + 2\theta_C\right)$$

$$M_{CD} = \frac{2EI}{4}\left(2\theta_C + 0 - \frac{3\Delta}{4}\right) = \frac{EI}{2}\left(2\theta_C - \frac{3\Delta}{4}\right)$$

$$M_{DC} = \frac{EI}{2}\left(\theta_C - 3\Delta/4\right)$$

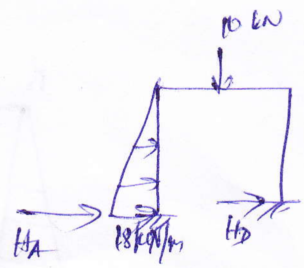
At joint B, $\Sigma M_B = 0$

$$\Rightarrow \frac{EI}{2}\left(2\theta_B - \frac{3\Delta}{4}\right) + 9.6 - 5 + \frac{EI}{2}\left(2\theta_B + \theta_C\right) = 0$$

$$4\theta_B + \theta_C - 0.75\Delta = -9.2 \quad (1)$$

(6)

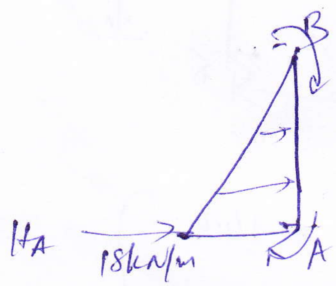
In overall frame, $\Sigma F_H = 0$



$$H_A + H_D = -\frac{1}{2} \times 18 \times 4$$

$$H_A + H_D = -36 \quad (A)$$

From column AB



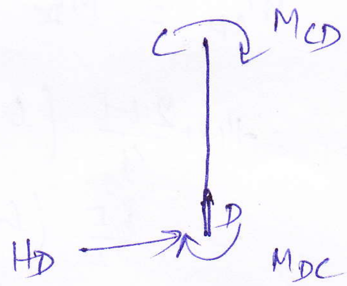
$\Sigma M_B = 0$

$$\Rightarrow M_{AB} + M_{BA} - H_A \times 4 - \frac{1}{2} \times 18 \times 4 \times \frac{2}{3} \times 4 = 0$$

$$4 \cdot H_A = M_{AB} + M_{BA} - 96$$

$$\boxed{H_{AB} = \frac{M_{AB} + M_{BA} - 96}{4}} \quad (B)$$

Consider FBD of CD



$\Sigma M_C = 0$

$$H_D = \frac{M_{CD} + M_{DC}}{4} \quad (C)$$

From (A), (B) & (C)

$$\frac{M_{AB} + M_{BA} - 96}{4} + \frac{(M_{CD} + M_{DC})}{4} = -36$$

$$M_{AB} + M_{BA} - 96 + M_{CD} + M_{DC} = -144$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} = -48$$

$$\Rightarrow -14.4 + \frac{EI}{2}(\theta_B - 0.75\Delta) + 9.6 + \frac{EI}{2}(2\theta_B - 0.75\Delta) + \frac{EI}{2}(2\theta_C - 0.75\Delta)$$
$$+ \frac{EI}{2}(\theta_C - 0.75\Delta) = -48$$

$$3\theta_B + 3\theta_C - 3\Delta = \frac{-86.4}{EI} \quad (3)$$

(07)

Eq. (1) − Eq^n (2) × 4

$$4\theta_B + \theta_C - 0.75\Delta = -\frac{9.2}{EI}$$
$$4\theta_B + 16\theta_C - 3\Delta = -\frac{40}{EI}$$
$$- \quad - \quad + \quad +$$

$$-15\theta_C + 2.25\Delta = \frac{30.8}{EI} \quad (4)$$

Eq^n (3) − Eq^n (2) × 3

$$3\theta_B + 3\theta_C - 3\Delta = -\frac{86.4}{EI}$$
$$3\theta_B + 12\theta_C - 2.25\Delta = -\frac{30}{EI}$$
$$- \quad - \quad + \quad +$$

$$-9\theta_C - 0.75\Delta = -\frac{56.4}{EI} \quad (5)$$

(4) + 3 × (5)

$$-15\theta_C + \cancel{2.25\Delta} = \frac{30.8}{EI}$$
$$-27\theta_C - \cancel{2.25\Delta} = -\frac{169.2}{EI}$$

$$-42\theta_C = -\frac{138.4}{EI}$$

$$\left.\begin{aligned} \theta_C &= \frac{3.295}{EI} \\ \Delta &= 35.657/EI \\ \theta_B &= \frac{3.562}{EI} \end{aligned}\right\}$$

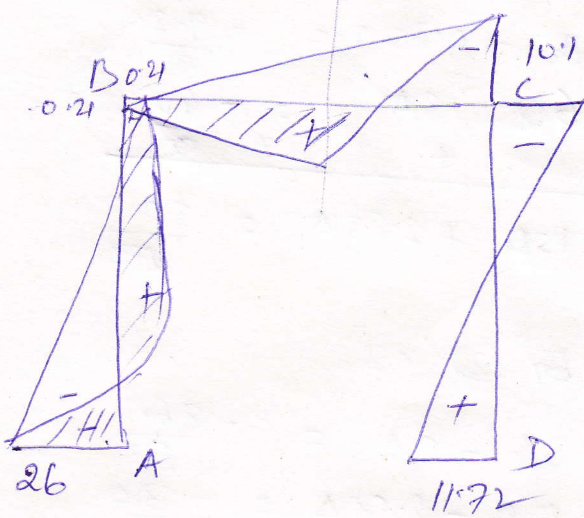
End moments are

$M_{AB} = -26.0$ kNm, $M_{BA} = -0.21$ kNm

$M_{BC} = 0.21$ kNm, $M_{CB} = 10.10$ kNm

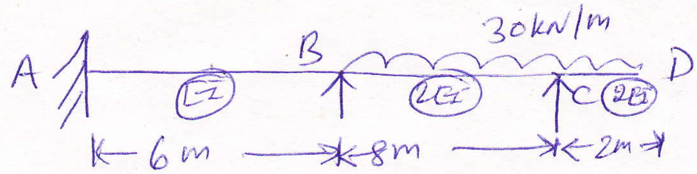
$M_{CD} = -10.10$ kNm, $M_{DC} = -11.72$ kNm

(08)



BMD in kNm

Q5.



$M_{CD} = -30 \times 2 \times 1 = -60 \text{ kNm}$

$M^F_{AB} = M^F_{BA} = 0, \quad M^F_{BC} = -\frac{30 \times 8^2}{12} = -160 \text{ kNm}$

$M^F_{CB} = +160 \text{ kNm}.$

$M_{AB} = \frac{2EI}{6}(2\theta_A + \theta_B) = \frac{EI}{3}(\theta_B)$

$M_{BA} = \frac{EI}{3}(2\theta_B)$

$M_{BC} = -160 + \frac{2EI \times 2}{8}(2\theta_B + \theta_c) = -160 + \frac{EI}{2}(2\theta_B + \theta_c)$

$M_{CB} = +160 + \frac{EI}{2}(\theta_B + 2\theta_c)$

Take $\Sigma M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0$

$\Rightarrow \frac{EI}{3}(2\theta_B) - 160 + \frac{EI}{2}(2\theta_B + \theta_c) = 0 \Rightarrow \frac{EI}{3}(2\theta_B) + \frac{EI}{2}(2\theta_B + \theta_c) = 160$

$\frac{4\theta_B + 6\theta_B + 3\theta_c}{6} = \frac{160}{EI} \Rightarrow 10\theta_B + 3\theta_c = \frac{960}{EI}$ (1)

Take $\Sigma M_C = 0 \Rightarrow M_{CB} + M_{CD} = 0$

$\Rightarrow 160 + \frac{EI}{2}(\theta_B + 2\theta_c) - 60 = 0$

$\Rightarrow \frac{EI}{2}(\theta_B + 2\theta_c) = -100 \Rightarrow \theta_B + 2\theta_c = -\frac{200}{EI}$ (2)

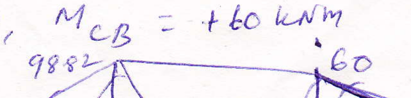
Eq^n (1) $- Eq^n$ (2) $\times 10 \Rightarrow$

$$\begin{aligned} \cancel{10\theta_B} + 3\theta_c &= \frac{960}{EI} \\ -\cancel{10\theta_B} \underset{-}{+20\theta_c} &= \underset{+}{-}\frac{2000}{EI} \\ \hline -17\theta_c &= \frac{2960}{EI} \end{aligned}$$

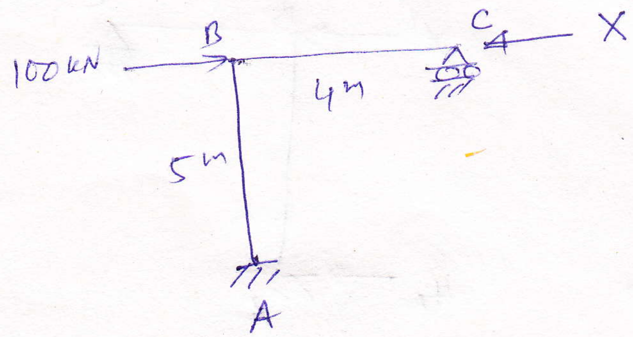
$\Rightarrow \theta_c = -\frac{174.12}{EI}, \quad \theta_B = \frac{148.24}{EI}$

Hence $M_{AB} = 49.41 \text{ kNm}, \; M_{BA} = 98.82 \text{ kNm}$

$M_{BC} = -98.82 \text{ kNm}, \; M_{CB} = +60 \text{ kNm}$

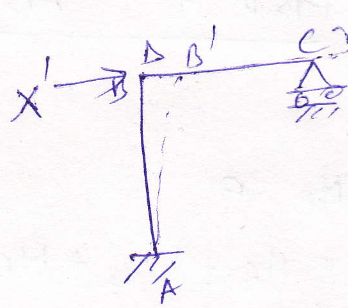


Q6



All FEM = 0 in I - Stage Moment distribution hence
$X = 100$ kN.

II - Stage Moment distribution



$M^F_{AB} = M^F_{BA} = -6\frac{EI\Delta}{5^2}$

$M^F_{BC} = M^F_{CB} = 0$

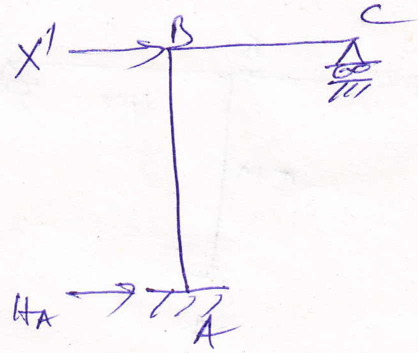
Take $EI\Delta = 100$, $M^F_{AB} = M^F_{BA} = -24$

Moment distribution

Joint	A	B		C
Member	AB	BA	BC	CB
R.S.	I/5	I/5	$\frac{3}{4}\times\frac{I}{4}$	I/4
DF	0	0.516	0.484	1
FEM	−24	−24	0	0
Bal	0	12.38	11.62	0
COF	6.19	0	0	0
Bal	0	0	0	0
				0
End moments	−17.81	−11.62	11.62	

(11)

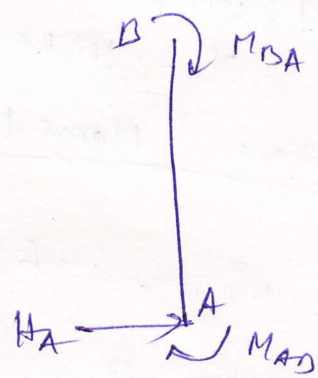
Consider FBD of overall frame.



$\Sigma F_H = 0$

$X' + H_A = 0$

FBD of AB



$\Sigma M_B = 0$

$-5 H_A + M_{AB} + M_{BA} = 0$

$H_A = \dfrac{M_{AB} + M_{BA}}{5} = -5.89\ kN\ (\leftarrow)$

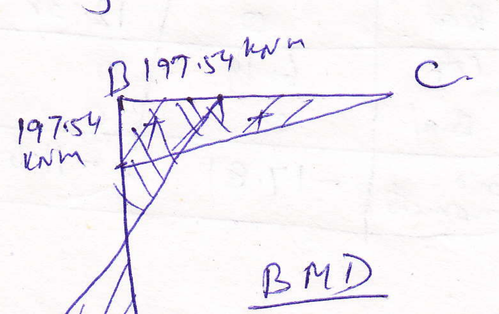
$X' = 5.89\ kN \rightarrow$

$MF = \dfrac{X}{X'} = \dfrac{100}{5.89} = 16.99 \approx 17$

Final end moments

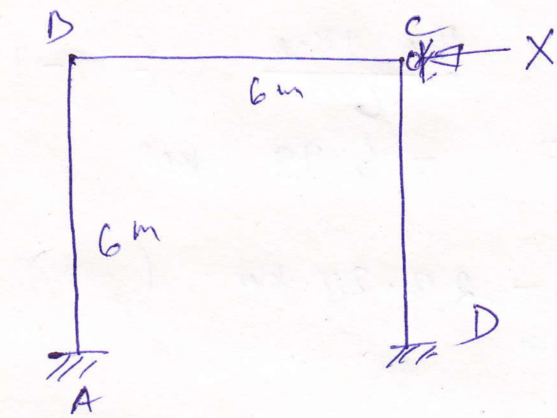
$$\left.\begin{aligned} M_{AB} &= -302.77 \\ M_{BA} &= -197.54 \\ M_{BC} &= +197.54 \end{aligned}\right\} kNm.$$

$M_{CB} = 0$



BMD

Q7



$\theta_A = -0.002$, $\Delta_{BC} = -6\ mm$

$EI = 5 \times 10^{10}\ kN\,mm^2 = 5 \times 10^{10} \times 10^{-6} = 5 \times 10^4\ kNm^2$

Nonsway moment distribution

$M^F_{AB} = -\frac{4EI\theta_A}{L} = -\frac{4 \times 5 \times 10^4 \times 0.002}{6} = -66.67\ kNm$

$M^F_{BA} = -\frac{2EI\theta_A}{L} = -33.33\ kNm$

$M^F_{BC} = -\frac{6EI\Delta_{BC}}{L^2} = +\frac{6 \times 5 \times 10^4 \times 6 \times 10^{-3}}{6^2}$

$= 50\ kNm$

$M^F_{CB} = +50\ kNm$, $M^F_{CD} = M^F_{DC} = 0$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
R.S.	I/6	I/6	I/6	I/6	I/6	I/6
DF	0	0.5	0.5	0.5	0.5	0
FEM	-66.67	-33.33	50	50	0	0
Bal	0	-8.34	-8.34	-25	-25	0
COF	-4.17	0	-12.5	-4.17	0	-12.5
Bal	0	+6.25	6.25	+2.09	2.09	0
COF	3.13	0	1.05	3.13	0	1.05
Bal	0	-0.53	-0.53	-1.57	-1.57	0

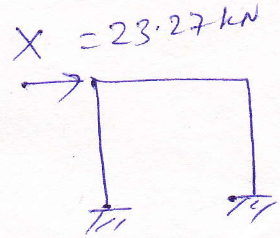
(13)

Shear eqⁿ $\quad H_A + H_D = X \quad$ (A)

$$H_A = \frac{M_{AB} + M_{BA}}{6} = -17.28 \text{ kN}$$

$$H_D = -5.99 \text{ kN}$$

$$X = -23.27 \text{ kN} \quad (\rightarrow)$$



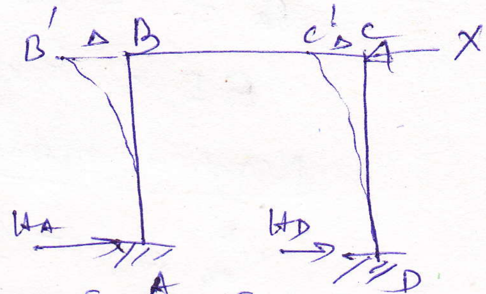
Non-sway moment distribution

$$M^F_{AB} = +\frac{6EI\Delta}{L^2} = \frac{6EI\Delta}{36} = M^F_{BA}$$

$$M^F_{BC} = M^F_{CB} = 0$$

$$M^F_{CD} = M^F_{DC} = \frac{6EI\Delta}{36}$$

Take $EI\Delta = 72$, $\quad M^F_{AB} = M^F_{BA} = M^F_{CD} = M^F_{DC} = 12$

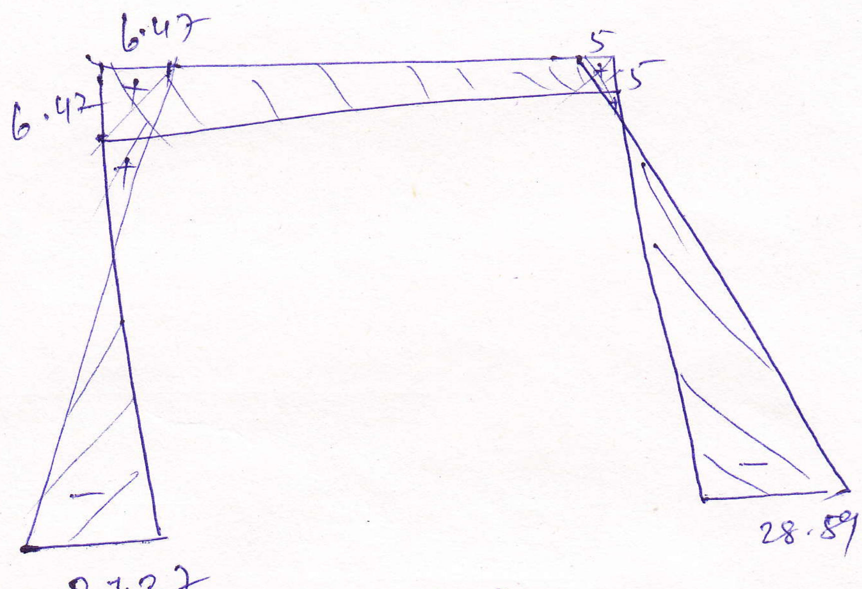


Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
RS	I/6	I/6	I/6	I/6	I/6	I/6
DF	0	0.5	0.5	0.5	0.5	0
FEM	12	12	0	0	12	12
Bal	0	-6	-6	-6	-6	0
COF	-3	0	-3	-3	0	-3
Bal	0	1.5	1.5	1.5	1.5	0
COF	0.75	0	0.75	0.75	0	0.75
Bal	0	-0.375	-0.375	-0.375	-0.375	0
End moments	9.75	7.125	-7.125	-7.125	7.125	9.75

$H_A = 2.81 \ (\rightarrow) \qquad H_D = 2.81$ kN

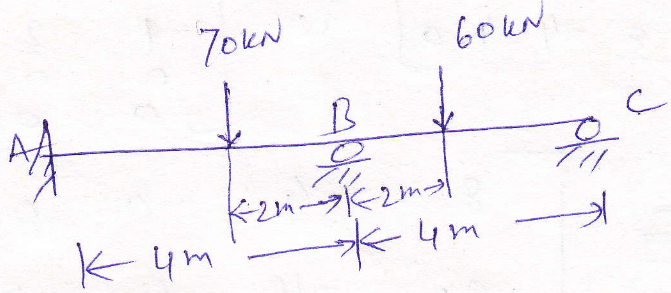
End moments

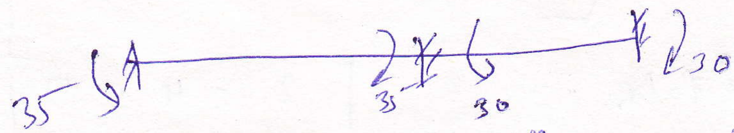
$$\left.\begin{aligned} M_{AB} &= -27.37 \\ M_{BA} &= -6.47 \\ M_{BC} &= 6.47 \\ M_{CB} &= -5.0 \\ M_{CD} &= 5.0 \\ M_{DC} &= 28.89 \end{aligned}\right\} \text{KNM}$$

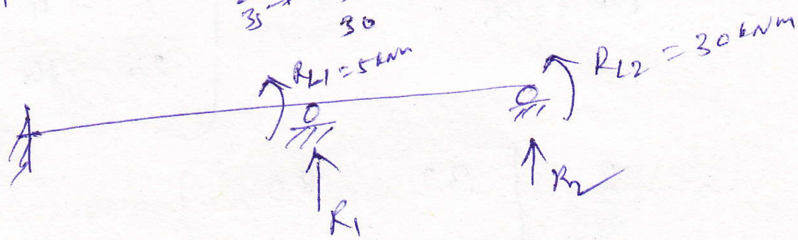


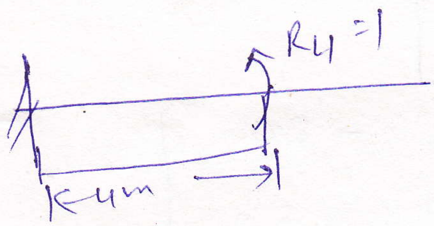
BMD
(KNM)

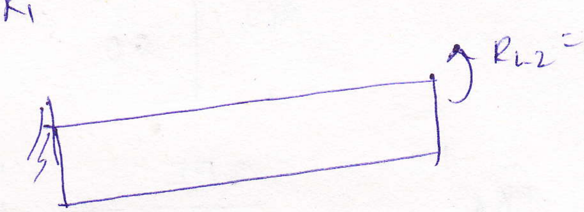
Q8

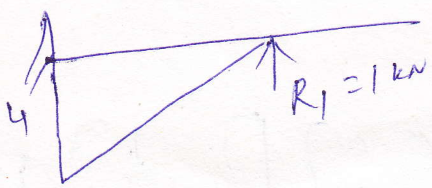


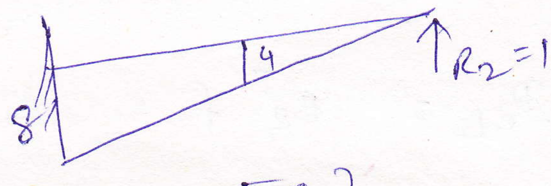












$[D] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Cor $D_1 = 0, \; D_2 = 0,$

$R_L = \begin{bmatrix} 5 \\ 30 \end{bmatrix}, \qquad q_M = \begin{bmatrix} -35 \\ +35 \\ -30 \\ +30 \end{bmatrix}$

$$B = \begin{array}{cc|cc} B_L & & B_R & \\ \end{array}$$
$$B = \left[\begin{array}{cc|cc} 1 & 1 & 4 & 8 \\ -1 & -1 & 0 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & -1 & 0 & 0 \end{array}\right]$$

$\begin{bmatrix} 2 & -1 & 0 & 0 \end{bmatrix}$ $\quad 2 \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \end{bmatrix}$

$$B_R^T f = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 8 & -4 & 4 & 0 \end{bmatrix} \frac{2}{3EI} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$= \frac{2}{3EI} \begin{bmatrix} 8 & -4 & 0 & 0 \\ 20 & -16 & 8 & -4 \end{bmatrix}$$

$$F_{RR} = B_R^T f B_R = \frac{2}{3EI} \begin{bmatrix} 8 & -4 & 0 & 0 \\ 20 & -16 & 8 & -4 \end{bmatrix} \begin{bmatrix} 4 & 8 \\ 0 & -4 \\ 0 & 4 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{2}{3EI} \begin{bmatrix} 32 & 80 \\ 80 & 256 \end{bmatrix}$$

$$F_{RR}^{-1} = \frac{3EI}{3584} \begin{bmatrix} 256 & -80 \\ -80 & 32 \end{bmatrix}$$

$$D_{RL} = B_R^T f B_L R_L$$

$$= \frac{2}{3EI} \begin{bmatrix} 8 & -4 & 0 & 0 \\ 20 & -16 & 8 & -4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 30 \end{bmatrix}$$

$$= \frac{2}{3EI} \begin{bmatrix} 420 \\ 1620 \end{bmatrix}$$

$$\begin{bmatrix} 256 & -80 \\ -80 & 32 \end{bmatrix} *$$

<u>case I</u>

$$R = F_{RR}^{-1} \left[D - D_{RL} \right] = \frac{3EI}{3584} \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \frac{2}{3EI} \begin{pmatrix} 420 \\ 1620 \end{pmatrix} \right]$$

$$= \frac{1}{1792} \begin{bmatrix} 256 & -80 \\ -80 & 32 \end{bmatrix} \begin{bmatrix} -420 \\ -1620 \end{bmatrix} = \begin{bmatrix} 12.32 \\ -10.18 \end{bmatrix}$$

$$q_0 = [B]\begin{bmatrix} R_L \\ R \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 4 & 8 \\ -1 & -1 & 0 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 30 \\ 12.32 \\ -10.18 \end{bmatrix}$$

$$= \begin{bmatrix} 2.84 \\ 5.72 \\ -10.72 \\ -30 \end{bmatrix}$$

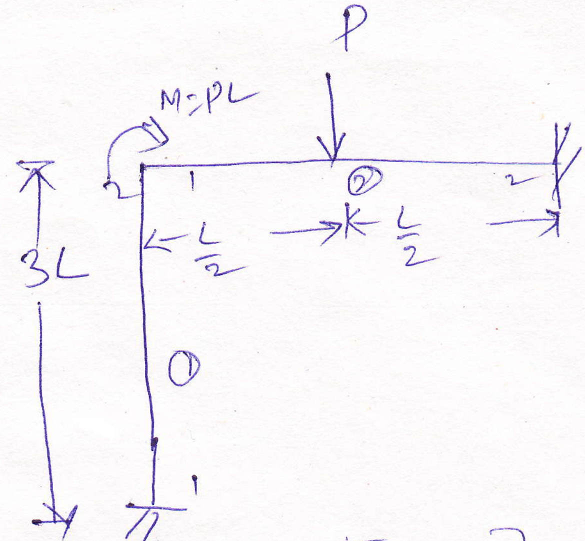
$$q = q_M + q_0 = \begin{bmatrix} -35 \\ +35 \\ -30 \\ +30 \end{bmatrix} + \begin{bmatrix} 2.84 \\ 5.72 \\ -10.72 \\ -30 \end{bmatrix}$$

$$q = \begin{bmatrix} -32.16 \\ 40.72 \\ -40.72 \\ 0 \end{bmatrix} \text{ kNm}$$

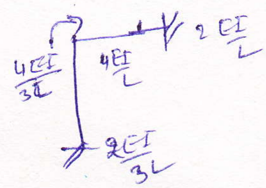
Similarly case II should be determined

where $D = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -350 \\ -250 \end{bmatrix}$

Q9



$q_D = [PL]$, $\quad q_M = \frac{PL}{8}\begin{bmatrix} 0 \\ 0 \\ -1 \\ +1 \end{bmatrix}$, $\quad q_L = \frac{PL}{8}[-1]$



$K = \cancel{4} \frac{4EI}{L}\left(1 + \frac{1}{3}\right)$

$= \frac{4EI}{L}\left(\frac{4}{3}\right) = \frac{16EI}{3L}$

$K^{-1} = \frac{3L}{16EI}[1]$

$K_M = \frac{EI}{L}\begin{bmatrix} \frac{2}{3} \\ 4/3 \\ 4 \\ 2 \end{bmatrix}$

$= \frac{EI}{3L}\begin{bmatrix} 2 \\ 4 \\ 12 \\ 6 \end{bmatrix}$

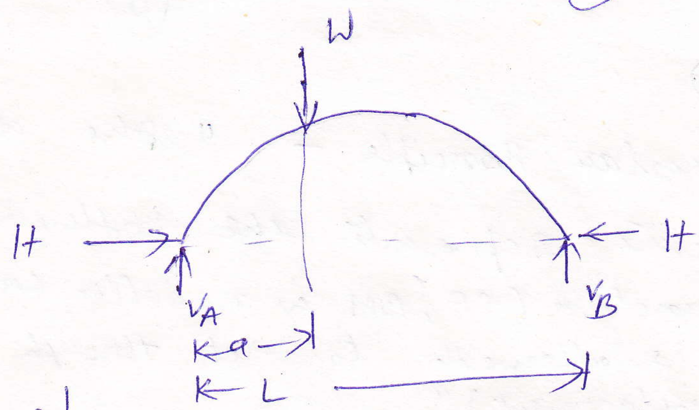
$D = K^{-1}[q_D - q_L] = \frac{3L}{16EI}[1]\left[PL + \frac{PL}{8}\right]$

$= \frac{27PL^2}{128EI}$

$q = q_M + K_M \cdot D = \frac{PL}{8}\begin{bmatrix} 0 \\ 0 \\ -1 \\ +1 \end{bmatrix} + \frac{EI}{3L}\begin{bmatrix} 2 \\ 4 \\ 12 \\ 6 \end{bmatrix} * \frac{27PL^2}{128EI}[1]$

$PL\left[\; 9/64 \;\right]$

Q10



$W = 60 kN$
$h = 6 m$
$L = 40 m$
$a = 12 m$

$$H = \frac{\int_0^L \frac{M_s y\, dx}{EI_c}}{\int_0^L \frac{y^2 dx}{EI_c}}$$

$$H = \frac{5 w a (l-a)(l^2 + al - a^2)}{8 h l^3}$$

$$= \frac{5 \times 60 \times 12 (40-12)(40^2 + 12 \times 40 - 12^2)}{8 \times 6 \times 40^3}$$

$$\cancel{= 63.29 kN} \quad = 63.53 kN$$

$$y = \frac{4h}{l^2}(lx - x^2) \quad , x = 12m, \quad y = 5.04m$$

$$M_{x=12m} = V_A \cdot 12 - Hy$$

$$= \frac{60(40-12)}{40} \times 12 - 63.53 \times 5.04$$

$$= 183.81 kNm$$

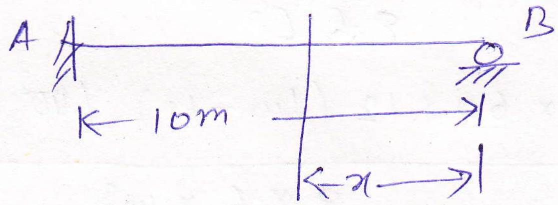
(20)

Q 11

(a)

Mulla Breslau Principle – " The deflected shape of the structure represents the influence line for any stress function (SF, BM or reaction component), if the function is allowed to act through a small distance (unit displacement)".

(b)



$$M_A = \frac{x}{2}\left(\frac{x^2}{l^2} - 1\right)$$

x (m)	0	1.25	2.5	3.75	5	6.25	7.5	8.75	10
M_A	0	−0.615	−1.172	−1.611	−1.875	−1.904	−1.640	−1.0225	0

When W = 100 kN at x = 5 m

(BM) at A = −1.875 × 100 = −187.5 kNm